Section 10.3

Vector FRQ Examples

Chapter 10 AP Packet #48:

An object moving along a curve in the xy-plane is at position $(x(t), y(t))$ at time t with

$$\frac{dx}{dt} = \arctan\left(\frac{t}{1+t}\right) \text{ and } \frac{dy}{dt} = \ln\left(t^2+1\right)$$

for $t \geq 0$. At time $t = 0$, the object is at position $(-3, -4)$. (Note: $\tan^{-1} x = \arctan x$)

(a) Find the speed of the object at time $t = 4$.

$$\sqrt{(x'(4))^2 + (y'(4))^2} = 2.912$$

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for $t \geq 0$. At time $t = 0$, the object is at position $(-3, -4)$. (Note: $\tan^{-1} x = \arctan x$)

(b) Find the total distance traveled by the object over the time interval $0 \leq t \leq 4$.

$$\int_0^4 \sqrt{(x'(t))^2 + (y'(t))^2}\, dt = 6.423$$

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for $t \geq 0$. At time $t = 0$, the object is at position $(-3, -4)$. (Note: $\tan^{-1} x = \arctan x$)

(c) Find $x(4)$.

$$x(4) = \underbrace{x(0)}_{-3} + \int_0^4 x'(t)\,dt = -.892$$

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for $t \geq 0$. At time $t = 0$, the object is at position $(-3, -4)$. (Note: $\tan^{-1} x = \arctan x$)

(d) For $t > 0$, there is a point on the curve where the line tangent to the curve has slope 2. At what time t is the object at this point? Find the acceleration vector at this point.

$$\frac{y'}{x'} = 2 \Rightarrow \frac{\ln(t^2+1)}{\tan^{-1}\left(\frac{t}{1+t}\right)} - 2 = 0$$

Solve in

Function mode

$$t = 1.35766$$

$$a(1.35766) = \langle x''(1.35766), y''(1.35766) \rangle$$

$$= \langle .135, .955 \rangle$$

Chapter 10 AP Packet #46:

An object moving along a curve in the xy-plane is at position $(x(t), y(t))$ at time t, where

$$\frac{dx}{dt} = \sin^{-1}\left(1 - 2e^{-t}\right) \text{ and } \frac{dy}{dt} = \frac{4t}{1+t^3}$$

for $t \geq 0$. At time $t = 2$, the object is at the point $(6, -3)$. (Note: $\sin^{-1} x = \arcsin x$)

(a) Find the acceleration vector and the speed of the object at time $t = 2$.

$$a(2) = \langle x''(2), y''(2) \rangle$$

$$= \langle .3956, -.7407 \rangle$$

SPEED: $\sqrt{(x'(2))^2 + (y'(2))^2}$

$$= 1.2075 \ldots$$

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for $t \geq 0$. At time $t = 2$, the object is at the point $(6, -3)$. (Note: $\sin^{-1} x = \arcsin x$)

(b) The curve has a vertical tangent line at one point. At what time t is the object at this point?

$x' = 0$

$\sin^{-1}(1-2e^{-t}) = 0$

USE CALCULATOR (FXN MODE)

$t = .693$

$1-2e^{-t} = \sin(0) = 0$

$1 = 2e^{-t}$

$\frac{1}{2} = e^{-t}$

$\ln(1/2) = -t$

$t = -\ln(1/2) = \ln(2)$

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for $t \geq 0$. At time $t = 2$, the object is at the point $(6, -3)$. (Note: $\sin^{-1} x = \arcsin x$)

(c) Let $m(t)$ denote the slope of the line tangent to the curve at the point $(x(t), y(t))$. Write an expression for $m(t)$ in terms of t and use it to evaluate $\lim_{t\to\infty} m(t)$.

$$m(t) = \frac{y'(t)}{x'(t)} = \frac{\frac{4t}{1+t^3} \to 0}{\sin^{-1}(1-2e^{-t} \to 0)} \qquad \lim_{t\to\infty} m(t) = \frac{0}{\pi/2} = 0$$

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for $t \geq 0$. At time $t = 2$, the object is at the point $(6, -3)$. (Note: $\sin^{-1} x = \arcsin x$)

(d) The graph of the curve has a horizontal asymptote $y = c$. Write, but do not evaluate, an expression involving an improper integral that represents this value c.

$$y(\infty) = \boxed{y(2) + \int_2^\infty y'(t)\,dt}$$

$y(e)$ e

$$\lim_{t\to\infty} y(t)$$

Classwork:

AP Packet #44

Homework:

AP Packet #45, 47, 49